BUBBLE GROWTH IN LIQUIDS

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UDC 536.423.1

Based on the first law of variable mass thermodynamics, a relation for the rate of bubble growth is obtained which is in good agreement with experimental data particularly for large Jakob numbers.

Existing theories of bubble growth [1-8] give satisfactory agreement with experimental data only for Jakob numbers less than 100 (see [9] for a detailed analysis). The results obtained below increase this range to Ja = 500.

We consider a bubble on a heat-emitting surface. The equation for the first law of thermodynamics for the bubble volume, assuming the vapor obeys the gas laws, can be written as follows [10]:

$$\frac{dp}{d\tau} = \frac{k-1}{V}\left(\frac{dQ}{d\tau} + i'G' - i''G'' - \frac{k}{k-1}\,p\,\frac{dV''}{d\tau}\right). \tag{1}$$

The pressure p in the bubble is determined from the well-known formula [11]

$$p = p_s + \frac{2\sigma}{R}. \tag{2}$$

The volume of a bubble "sitting" on a plane is

$$V = \frac{4}{3}\pi R^3 f_V. \tag{3}$$

Here

$$f_V = \frac{1}{2}\left[1 + \frac{1}{2}\cos\theta\,(2 + \sin^2\theta)\right].$$

The heat transferred to the bubble is made up of the heat arriving from the liquid, which is (see [6])

$$\frac{dQ'}{d\tau} = \frac{\lambda'\Delta T}{\sqrt{\pi a'\tau}}\,4\pi R^2 f_s, \tag{4}$$

and the heat received by the vapor from the base of the bubble (through a microlayer)

$$\frac{dQ''}{d\tau} = \pi R^2 \sin^2\theta\, q. \tag{5}$$

Here

$$f_s = \frac{1}{2}(1 + \cos\theta).$$

The heat supplied to the bubble is used up in the evaporation of a mass G of the liquid, i.e., the difference between the incoming energy of mass i'G' and its consumption i"G" is

$$i'G' - i''G'' = -r\rho''\frac{dV}{d\tau}, \tag{6}$$

300-Year-Reunion-of-the-Ukraine-with-Russia State University, Dnepropetrovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 18, No. 5, pp. 844-848, May, 1970. Original article submitted February 27, 1969.

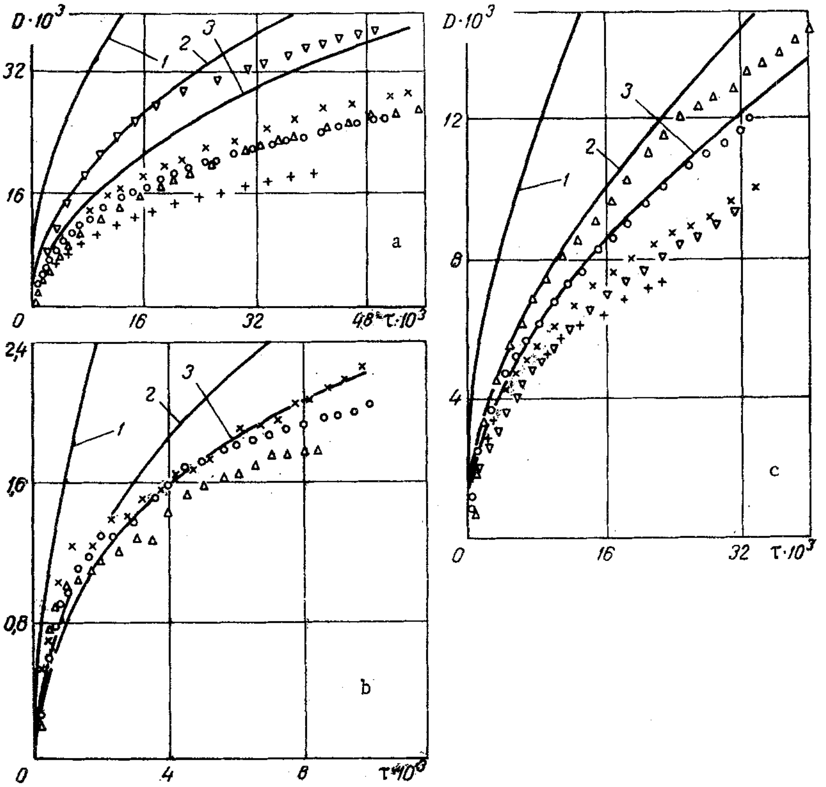

Fig. 1. Comparison of theoretical and experimental results (D, m; τ, sec): a) water at a pressure p = 98 mm Hg, $\Delta T = 15°$, Ja = 301, $q = 3.73 \cdot 10^4$ W/m^2; b) n-pentane, p = 524 mm Hg, $\Delta T = 28°$, Ja = 52.8, $q = 3.62 \cdot 10^4$ W/m^2; c) methanol, p = 204 mm Hg, $\Delta T = 26.5°$, Ja = 140.5, $q = 2.8 \cdot 10^4$ W/m^2.

and on completion of the work of expansion, is proportional to the increase in bubble volume after subtraction of the volume occupied by the evaporated liquid, i.e.,

$$\frac{dV''}{d\tau} = \frac{dV}{d\tau} - \frac{dV'}{d\tau} = \left(1 - \frac{\rho''}{\rho'}\right)\frac{dV}{d\tau}. \tag{7}$$

It must be noted that neglect of the work of expansion in [12] leads to an overestimate in the theoretical results of all papers using the Bosnyakovic equation, as we shall see below, particularly for Ja > 100.

Considering Equations (2)-(7), it is easy to transform the original Eq. (1) to the following:

$$\left[1 + \frac{k}{k-1} f_\rho \frac{p_s}{r\rho''} + \frac{2\sigma}{3(k-1)r\rho''}(3f_\rho k - 1)\frac{1}{R}\right]\frac{dR}{d\tau} = f_\theta\left(\frac{\lambda'\Delta T}{r\rho''\sqrt{\pi a'\tau}} + f_q\frac{q}{r\rho''}\right), \tag{8}$$

the integration of which offers no special difficulty:

$$\left(1 + \frac{k}{k-1} f_\rho N_1\right)(R - R_0) + \frac{2}{3}\frac{(3f_\rho k - 1)}{k-1} N_2 R_0 \ln\frac{R}{R_0} = f_\theta \,\mathrm{Ja}\left(\frac{2}{\sqrt{\pi}}\sqrt{a'\tau} + f_q N_3 \tau\right). \tag{9}$$

Here

$$N_1 = \frac{p_s}{r\rho''}; \quad N_2 = \frac{\sigma}{r\rho'' R_0}; \quad N_3 = \frac{q}{\Delta T c'\rho'};$$

$$\mathrm{Ja} = \frac{\Delta T c'\rho'}{\rho'' r}; \quad f_q = \frac{1}{2}(1 - \cos\theta); \quad f_\theta = \frac{f_s}{f_V}.$$

If one uses the well-known relation between the heat flux q at vaporization and the temperature drop ΔT

$$q = A\Delta T^n,$$

it is easy to show that the quantity $N_3 = \omega_\alpha$ is associated with the Jakob number:

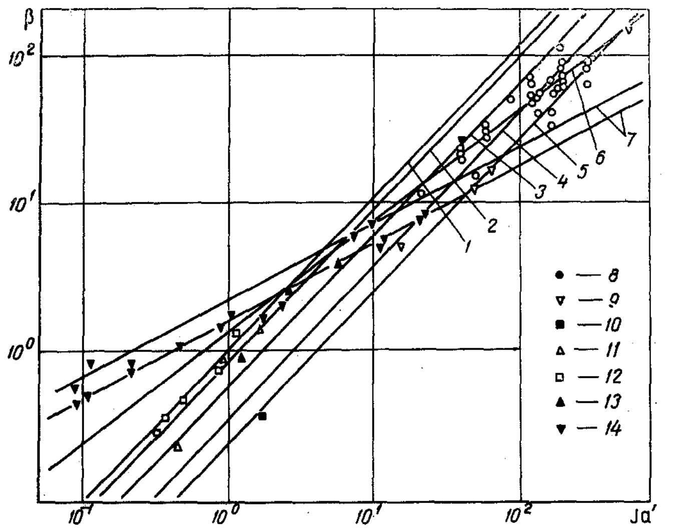

Fig. 2. β as a function of Ja': theoretical curves; 1) Plesset and Zwick [4]; 2) Forster and Zuber [7]; 3) Fritz and Ende [5]; 4) Eq. (13); 5) Van Stralen [14]; 6) empirical curve of Cole and Shulman, $R = 2.5\ \mathrm{Ja}\sqrt{a'\tau}$ [9]; 7) D. A. Labuntsov [1-3] (upper curve for proportionality constant of 10 [2], lower for 6 [3]); experimental points: 8) Cole and Shulman [9]; 9) Van Stralen [11]; 10) R. Semeria [14]; 11) Dergarabedian [14]; 12) Van Stralen [14]; 13) Benjamin–Westwater [14]; 14) D. A. Labuntsov [3].

$$w_\alpha = B\,\mathrm{Ja}^{n-1},$$

with the quantity $B = A(r\rho'')^{n-1}/(\rho'c')^n$ being determined by the kind of liquid, and surface and temperature (pressure) conditions.

Equation (9) is easily brought into dimensionless form

$$\left(1+\frac{k}{k-1}f_\rho N_1\right)\left(\frac{\bar{R}}{\bar{R}_0}-1\right)+\frac{2}{3}\frac{(3f_\rho k-1)}{k-1}N_2\ln\frac{\bar{R}}{\bar{R}_0}=\frac{2}{\sqrt{\pi}}\varphi_\theta\frac{\sqrt{a'\tau}}{R_0}\frac{\mathrm{Ja}}{\bar{R}_0}+\varphi_\theta\varphi_q\left(\frac{\sqrt{a'\tau}}{R_0}\right)^2\mathrm{Pe}_*.$$

We used the experimental data in [9] to verify this equation. A comparison of calculations using Eq. (9) (curve 3) with experimental results is shown in Fig. 1, which also gives the calculated results of Cole and Shulman based on the Zuber equation [6] (curve 1) and the curve based on the equation $R = \sqrt{\pi}\,\mathrm{Ja}\sqrt{a'\tau}/2$ (curve 2).†

As is clear from Fig. 1, calculations based on Eq. (9) give more accurate results for different Jakob numbers and different materials (water, n-pentane, and methanol).

The second term on the left side of Eq. (9) can be neglected and the coefficient f_ρ taken equal to 1 when $2\sigma/R \ll p_s$ and $\rho''/\rho' \ll 1$ (at pressures sufficiently removed from the critical value) and also for rough calculations. Using the relation for k (see [13])

$$k=\left[1-\frac{2p_s}{r\rho''}\right]^{-1}$$

Eq. (9) then reduces to the form

$$R = R_0 + \frac{2}{3}f_\theta\,\mathrm{Ja}\left(\frac{2}{\sqrt{\pi}}\sqrt{a'\tau}+f_qN_3\tau\right). \tag{10}$$

We then obtain the following result for the case of bubble growth within the liquid volume ($\theta = 0$; $q = 0$; $f_\theta = 1$):

† Curves calculated by other methods are not shown because they give a considerably greater overestimate (see below with respect to this point).

$$R = \frac{4}{3\sqrt{\pi}} \mathrm{Ja} \sqrt{a'\tau}. \tag{11}$$

For overall analysis and comparison of the results with experiment in dimensionless form, we write this equation in the form obtained by Scriven [8]:

$$R = 2\beta \sqrt{a'\tau}, \tag{12}$$

where

$$\beta = \frac{2}{3\sqrt{\pi}} \mathrm{Ja}'. \tag{13}$$

In our case, Ja' = Ja.

Curves for $\beta = \beta(\mathrm{Ja}')$ are shown in Fig. 2, which also shows calculated results based on the equation of Labuntsov (curve 7) and those obtained with Eq. (13) (curve 4); as can easily be verified, the latter gives better agreement with the experimental points.

NOTATION

k	is the adiabatic index for vapor (see, for example [15]);
i	is the enthalpy;
G	is the second change of mass;
τ	is the time;
p_s	is the saturated vapor pressure;
R	is the radius of bubble;
σ	is the surface tension coefficient;
θ	is the boundary angle;
λ	is the thermal conductivity;
a	is the thermal diffusivity;
ΔT	is the superheating relative to saturation temperature;
q	is the heat flux;
ρ	is the density;
r	is the heat of evaporation;
c	is the heat capacity;
R_0	is the initial radius;
A	is the constant depending on kind of liquid, state of heat transfer surface;
n	is the index depending on kind of liquid;

$Pe_* = qR_*/r\rho''a'$;
$R_* = \sqrt{\sigma/g(\rho'-\rho'')}$;
$R = R/R_*$.

Superscripts

' denotes to liquid;
'' denotes to vapour.

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